Can black holes be torn up by phantom dark energy in cyclic cosmology?

Xin Zhang

Department of Physics, College of Sciences, Northeastern University, Shenyang 110004, People's Republic of China and
Kavli Institute for Theoretical Physics China at the Chinese Academy of Sciences,
P.O.Box 2735, Beijing 100080, People's Republic of China

Infinitely cyclic cosmology is often frustrated by the black hole problem. It has been speculated that this obstacle in cyclic cosmology can be removed by taking into account a peculiar cyclic model derived from loop quantum cosmology or the braneworld scenario, in which phantom dark energy plays a crucial role. In this peculiar cyclic model, the mechanism of solving the black hole problem is through tearing up black holes by phantom. However, using the theory of fluid accretion onto black holes, we show in this paper that there exists another possibility: that black holes cannot be torn up by phantom in this cyclic model. We discussed this possibility and showed that the masses of black holes might first decrease and then increase, through phantom accretion onto black holes in the expanding stage of the cyclic universe.

The oscillating or cyclic model of the universe is an attractive idea in theoretical cosmology since it provides the universe with an infinite life satisfying, in some sense, human being's philosophical psychology of expecting eternalness. In cyclic cosmology, the universe oscillates through a series of expansions and contractions. The idea of an oscillating universe was first proposed by Tolman in the 1930's [1]. In recent years, Steinhardt, Turok and collaborators [2] proposed a cyclic model of the universe as an alternative to the inflation scenario, in which the cyclicity of the universe is realized in the light of two separated branes. In general, however, cyclic universe models confront two severe problems making the infinite cyclicity impossible. First, the black holes produced in the universe, which cannot disappear due to the Hawking area theorems, grow ever larger during subsequent cycles, and they eventually will occupy the entire horizon volume during a contracting phase so that calculations in cyclic models break down. The second problem is that the entropy of the universe increases from cycle to cycle due to the second law of thermodynamics, so that extrapolation into the past will lead back to an initial singularity.

Recently, a new version of oscillating cosmology [3, 4] (also dubbed "phantom bounce" in [3]) claimed that the problems of black holes and entropy puzzling cyclic models can be resolved by means of the peculiar characteristic of the phantom dark energy in the universe. Usually, the phantom energy density becomes infinite in a finite time, leading to the big-rip singularity [6]. However, we expect that an epoch of quantum gravity sets in before the energy density reaches infinity. Therefore, we arrive at the notion that quantum gravity governs the behavior of the universe both at the beginning and at the end of the expanding universe, where the energy density is enormously high. The high energy density physics may lead to modifications to the Friedmann equation; for example, it may introduce a negative ρ^2 term, such as in loop quantum cosmology [7] and braneworld scenario [8], which causes the universe to bounce when it is small, and to turn around when it is large. The cyclic scenario discussed in this paper is distinguished from the Steinhardt−Turok cyclic scenarios in that the phantom energy plays a crucial role. In such a cyclic cosmology, it was shown in [3] that black holes in the universe will be torn up by the phantom dark energy before the turnaround. Also, in [4] the authors claimed that at the turnaround of the cyclic cosmology both volume and entropy of our universe will decrease by a gigantic factor, while very many independent similarly small contracting universes will be spawned, thus resolving the entropy problem of the cyclic cosmology.[1]

The key idea for eliminating the black hole problem in cyclic cosmology is that any black holes formed in an expanding phase of the universe are torn apart by the phantom dark energy before they can create problems during contraction. However, the discussions on destruction of black holes in [3] are based on a rough evaluation. In general relativity, the source for a gravitational potential is the volume integral of $\rho + 3p$. Therefore, an object of radius R and mass M is pulled apart when $-(4\pi/3)(\rho + 3p)R^3 \sim M$. A black hole with radius $R = 2GM$ is thus torn up when the phantom energy density has climbed up to a value $\rho_{\rm bh} \sim (3/32\pi)(M^2G^3|1 + 3w|)^{-1}$, where G is the Newton gravitational constant, and $w = p/\rho < -1$ is the equation of state of the phantom dark energy. For ensuring that the black holes are destroyed before turnaround, one only needs $\rho_{\rm bh} < \rho_{\rm c}$, where $\rho_{\rm c}$ is the critical energy density in the cyclic model, namely the energy density corresponding to the turnaround (and bounce).

However, the destruction of black holes by phantom accretion is not an instantaneous behavior, it is a process actually. In a phantom dominated universe with "big rip", black holes can be torn up completely by phantom energy before the big rip, however, in such a cyclic cosmology caused by the "phantom bounce", whether or not black holes can be torn up by phantom energy should be investigated in detail by using the theory of dark energy accretion by black holes. We shall study in this paper the phantom accretion onto a black hole in the cyclic universe (in the expanding phase dominated by phantom component), and show that there exists the possibility that the mass of the black hole may decrease first, to a minimum, and increase then, until restoring the original mass value at the turnaround. Thus, actually, according to this possibility, phantom energy cannot help resolve the black hole problem in cyclic cosmology.

For the fluid accretion onto a black hole, Babichev et al.

[1] For a different viewpoint on this scenario, see [5].

have obtained a successful mechanism [9] in which, as a consequence of fluid accretion, the mass of the black hole changes at a rate $\dot{M} = 4\pi A M^2[\rho_\infty + p(\rho_\infty)]$, where A is a positive dimensionless constant, and ρ_∞ and $p(\rho_\infty)$ are the energy density and pressure of the fluid at the remote distance from the black hole, respectively. For the case of a wormhole, see [10]. Following the mechanism of Babichev et al., we shall study the phantom accretion of black hole in the cyclic cosmology in this paper.

Let us consider a modified Friedmann equation in which a ρ^2 term with negative sign is introduced due to some quantum gravity effects,

$$H^2 = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right), \tag{1}$$

where $H = \dot{a}/a$ is the Hubble parameter, and ρ_c is the critical energy density set by quantum gravity, distinguished from the usual critical density $3M_{\rm pl}^2 H^2$ (where $M_{\rm pl} = 1/\sqrt{8\pi G}$ is the reduced Planck mass). This modified Friedmann equation can be derived from the effective theory of loop quantum cosmology [7], and also from the braneworld scenario [8]. In loop quantum cosmology, the critical energy density can be evaluated as $\rho_c \approx 0.82\rho_{\rm pl}$, where $\rho_{\rm pl} = G^{-2} = 2.22\times10^{76}$ GeV^4 is the Planck density. In the braneworld scenario, $\rho_c = 2\sigma$, where σ is the brane tension, and a negative sign in Eq. (1) can arise from a second timelike dimension but that gives difficulties with closed timelike paths. In models motivated by the Randall−Sundrum scenario [11], the most natural energy scale of the brane tension is of the order of the Planck mass, but the problem can be generally treated for any value of $\sigma >$ TeV^4.

While we use loop quantum cosmology only as an example, we feel that it would be better to make some clarifications for the background for avoiding misunderstandings. For instance, the value for ρ_c given above is based on an ad hoc choice and not derived from a general setting. In general, there can be a different numerical factor, and the critical density can even depend on the scale factor depending on the precise underlying state [12]. Moreover, the evolution equation in the form (1) provides the correct effective theory only in the case of a stiff fluid (such as a free, massless scalar) which, strictly speaking, is not the case discussed here. In general, there are additional quantum corrections due to the interacting nature of the system. The latest status on this issue can be seen, e.g., in [12]. Therefore, it should be stressed that the simple equation (1) only serves as an example, and one should not indiscreetly say that this is the general situation.

In the regime very near the turnaround, the term $(1 - \rho/\rho_c)$ in (1) is almost zero, and one can expect that additional correction terms become very important. This is true for loop quantum cosmology, and for braneworlds one might similarly expect additional corrections due, e.g., to higher order terms in the string tension. Therefore, although the equation (1) is frequently used in the literature, the readers should be cautioned that the equations themselves might not give the full picture. In this paper, however, we only consider the simple case based on Eq. (1), i.e., we treat the problem phenomenologically, following the literature, such as [3, 4].

Such a modified Friedmann equation with a phantom energy component leads to a cyclic universe scenario in which the universe oscillates through a series of expansions and contractions [3, 4]. Phantom energy can dominate the universe today and drive the current cosmic acceleration [13]. Then, as the universe expands, it becomes more and more dominant and its energy density becomes very high. When the phantom energy density reaches the critical value ρ_c, the universe reaches a state of maximum expansion which we call "turnaround", and then begins to recollapse, according to the modified Friedmann equation. The contraction of the universe makes the phantom energy density dilute away and the matter density dominate. Once the universe reaches its smallest extent, the matter density hits the value of the critical density, the modified Friedmann equation leads to a "bounce", making the universe once again begin to expand. Note that both turnaround and bounce are nonsingular in this scenario.

In this paper, we only consider the high energy regime in the expanding branch, where phantom energy is overwhelming and the ρ^2 effect is prominent. Since in the high energy regime we have $\rho \gg \rho_{\rm today}$, we say $\rho_{\rm today} \sim 0$. Phantom dark energy is characterized by the parameter of equation of state $w = p/\rho < -1$ which is considered to be a constant in this paper for convenience. Combining the modified Friedmann equation (1) and the conservation law $\dot{\rho} + 3H(\rho + p) = 0$ yields

$$\dot{H} = -4\pi(\rho + p)\left(1 - \frac{2\rho}{\rho_c}\right), \tag{2}$$

where we have set $G = 1$ for convenience (this convention will be used hereafter). From Eqs. (1) and (2), we derive

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3}\left\{\rho\left(1 - \frac{\rho}{\rho_c}\right) + 3\left[p\left(1 - \frac{2\rho}{\rho_c}\right) - \frac{\rho^2}{\rho_c}\right]\right\}. \tag{3}$$

Comparing to the classical form of the equation, it is convenient to define the effective energy density and pressure

$$\rho_{\rm eff} = \rho\left(1 - \frac{\rho}{\rho_c}\right), \tag{4}$$

$$p_{\rm eff} = p\left(1 - \frac{2\rho}{\rho_c}\right) - \frac{\rho^2}{\rho_c}, \tag{5}$$

then Eq. (3) can be written as

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3}(\rho_{\rm eff} + 3p_{\rm eff}). \tag{6}$$

Also, we have $H^2 = 8\pi\rho_{\rm eff}/3$ and $\dot{H} = -4\pi(\rho_{\rm eff} + p_{\rm eff})$, obviously.

This means that all the quantum gravity effects can be attributed to the effective quantities such as $\rho_{\rm eff}$ and $p_{\rm eff}$. According to this perspective, classical general relativity (Einstein equation) can also be used to describe quantum cosmology, provided that all the quantum gravity effects could be effectively attributed to the energy-momentum tensor, i.e. the Einstein equation under such circumstances should be effectively written as $G_{\mu\nu} = 8\pi T^{\rm eff}_{\mu\nu}$. So, under this description,

the universe looks like filled with the effective fluid with $\rho_{\rm eff}$ and $p_{\rm eff}$. Using $\rho_{\rm eff}$ and $p_{\rm eff}$, we can effectively describe the behavior of the universe.

Given the effective energy density and pressure, the effective equation of state is defined naturally as

$$w_{\rm eff} = \frac{p_{\rm eff}}{\rho_{\rm eff}} = \frac{w(1-2x)-x}{1-x}, \tag{7}$$

where x is defined as dimensionless density, $x = \rho/\rho_c$, so we have $0 < x < 1$.

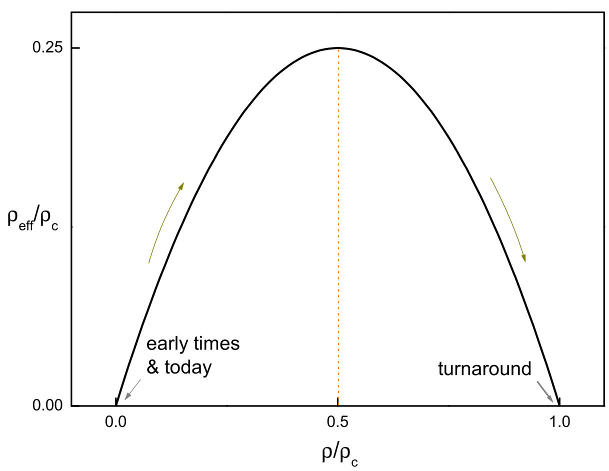

FIG. 1: Sketch map of the expanding phase of phantom dominated universe in the cyclic cosmology. In the expanding stage, the phantom energy density increases monotonously, whereas the effective energy density of the universe first increases and then decreases, implying an effective behavior of "quintom", due to the quantum gravity effects. The effective energy density $\rho_{\rm eff}$ arrives at its maximum, $\rho_{\rm eff}^{\rm max} = \rho_c/4$, when the phantom density reaches the half value of the critical density, $\rho = \rho_c/2$.

Albeit phantom energy density always increases monotonously with the expansion of the universe, the effective energy density, however, exhibits totally different behavior comparing to the phantom energy density. Fig. 1 plots the rewritten Eq. (4), $y = x(1-x)$, where $y = \rho_{\rm eff}/\rho_c$ and $x = \rho/\rho_c$. It is clear that the effective energy density $\rho_{\rm eff}$ first increases and then decreases, which implies that the effective behavior of the universe under the quantum gravity domination resembles a "quintom"[2] whose key feature is that its equation of state can evolve across the "cosmological constant boundary". One can check that $w_{\rm eff} < -1$ in the range $0 < x < 1/2$, and $w_{\rm eff} > -1$ within $1/2 < x < 1$, provided that $w < -1$. Hence, we learn that the place $x = 1/2$ plays the role of the "phantom divide" for the effective energy density of the universe. The dimensionless effective energy density y arrives at its maximum, $y_{\rm max} = 1/4$, when the dimensionless phantom density reaches the value $x = 1/2$.

[2] For quintom dark energy see [14], and for the detailed analysis for the effective quintom behavior in loop quantum cosmology see [15].

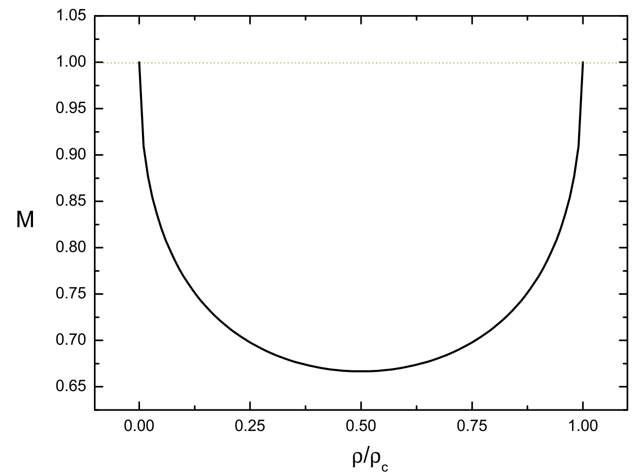

FIG. 2: The variation of black hole mass due to the phantom dark energy accretion in the cyclic cosmology. Here we show a simplified case, $M(x) = 1/(1+\sqrt{x(1-x)})$, where $x = \rho/\rho_c$, for illustration. In the cyclic universe dominated by phantom dark energy, the phantom accretion makes the black hole mass first decrease, to a minimum, then increase, until restoring its initial value at the turnaround.

Consider now the phantom dark energy accretion of a Schwarzschild black hole in such a cyclic universe. As shown above, at the level of effective theory, the quantum gravity effects make the phantom energy behave as a quintom energy in such a universe. Thus, at remote distances from the black hole (namely, on the cosmic scale), the universe looks like filled with the effective fluid with $\rho_{\rm eff}$ and $p_{\rm eff}$. That is to say, ρ_∞ behaves as $\rho_{\rm eff}$ when taking the quantum gravity effects into account. Hence, considering the resulting cosmological evolution in this model, we should replace ρ_∞ by $\rho_{\rm eff}$ (and replace $p(\rho_\infty)$ by $p_{\rm eff}$). However, in the vicinity of the black hole (namely, on the local scale), we assume that the dark energy is still described by ρ and p (note that here ρ and p are not quantities on the cosmic scale, so they are not homogeneous due to the influence of the black hole), since the physics on cosmic scale does not directly affect the physics on local scale. Following Babichev et al. [9], we can deal with the fluid accretion onto a black hole. First, the integration of the time component of the energy-momentum conservation law $T^{\mu\nu}_{;\nu} = 0$ gives the first integral of motion for the stationary spherically symmetric accretion:

$$(\rho+p)\left(1-\frac{2M}{r}+u^2\right)^{1/2}\frac{ur^2}{M^2} = B, \tag{8}$$

where $u = dr/ds$, r is the Schwarzschild radial coordinate, M is the mass of a black hole, and B is a constant. Next, using the projection of the energy-momentum conservation law on the four-velocity $u_\mu T^{\mu\nu}_{;\nu} = 0$, where $u^\mu = dx^\mu/ds$ is the fluid four velocity with $u^\mu u_\mu = 1$, one obtains the second integral of motion:

$$\frac{ur^2}{M^2}\exp\left[\int_{\rho_\infty}^{\rho}\frac{d\rho'}{\rho'+p(\rho')}\right] = -A, \tag{9}$$

where $u < 0$ stands for the case of inflow motion and A is a positive dimensionless constant. From (8) and (9), one can easily derives

$$(\rho + p)\left(1 - \frac{2M}{r} + u^2\right)^{1/2} \exp\left[\int_{\rho}^{\rho_\infty} \frac{d\rho'}{\rho' + p(\rho')}\right] = \rho_\infty + p(\rho_\infty). \tag{10}$$

The fluid accretion gives rise to that the mass of a black hole changes at a rate $\dot{M} = -4\pi r^2 T^r_0$. With the help of (9) and (10), one obtains [9]

$$\dot{M} = 4\pi A M^2[\rho_\infty + p(\rho_\infty)]. \tag{11}$$

This shows that the rate of change of the black hole mass is determined by the behavior of dark energy on cosmic scales.

As mentioned, the cosmological evolution behavior of phantom energy in the cyclic cosmology mimics the behavior of a quintom energy due to the modified Friedmann equation $H^2 \sim \rho_{\rm eff}$. Thus, it is obvious that ρ_∞ and $p(\rho_\infty)$ in Eq. (11) should be replaced by $\rho_{\rm eff}$ and $p_{\rm eff}$.[3] Therefore, following the procedure of Babichev et al. [9], one can write down the change rate of the black hole mass,

$$\dot{M} = 4\pi A M^2(\rho_{\rm eff} + p_{\rm eff}). \tag{12}$$

From this equation, it is clear that the accretion of the phantom energy with $w < -1$ in a cyclic universe cannot ensure the monotonously diminishing of the black hole mass since the effective energy density behaves as a quintom, i.e., $1 + w_{\rm eff}$ evolves from values smaller than 0 to larger than 0. Replacing d/dt in (12) with $-\sqrt{3}(1 + w)\sqrt{\rho_{\rm eff}}\rho d/d\rho$, one obtains the following expression,

$$\frac{dM}{M^2} = -\frac{4\pi A(1 + w_{\rm eff})\sqrt{\rho_{\rm eff}}}{\sqrt{3}(1 + w)\rho} d\rho. \tag{13}$$

By integrating this equation, we obtain

$$M = \frac{M_0}{1 + CM_0}, \tag{14}$$

where M_0 is the initial mass of the black hole, and $C = 8\pi A\sqrt{\rho_c x(1 - x)/3}$. Note that here the initial condition is chosen as $M|_{x_i=0} = M_0$. Equation (14) explicitly indicates that, in the cyclic universe caused by phantom bounce, through the phantom accretion, black hole mass will decrease first, and then increase until restoring its initial mass at the turnaround. The minimum value of the black hole mass, $M_{\rm min} = M_0/(1 + 4\pi A\sqrt{\rho_c/3}M_0)$, happens at $x = 1/2$. For illustrating the black hole mass variation, we take a simple case as example, i.e., $M_0 = 1$ and $8\pi A\sqrt{\rho_c/3} = 1$, as shown in Fig. 2. Therefore, so far, we learn, from the analysis of the fluid accretion of black holes, that masses of black holes are not diminishing monotonously by phantom accretion in the cyclic cosmology, i.e., in such a cyclic universe black holes cannot be torn up by phantom dark energy.

Undoubtedly, in the usual phantom dominated universe with big rip, phantom accretion onto black holes will always diminish black hole masses. In this case, the solution of mass variation of black hole is also in the same form as (14) but where $C = 8\pi A\sqrt{\rho/3}$.[4] Obviously, when the universe goes towards the big-rip, $\rho \to \infty$, we have $M \to 0$, implying that black holes are torn up by phantom near the big rip. However, in the cyclic universe discussed in this paper, the phantom dark energy behaves like a quintom dark energy effectively, due to the quantum gravity effects, so that the phantom accretion by black holes makes the black hole masses first decrease and then increase. Nevertheless, it should also be emphasized that the result obtained in this paper is heavily based on the work of Babichev et al. [9].

One may notice, however, that there are some problems in the calculations of Babichev et al. [9]. First, in their work, the metric used is asymptotically flat (actually, the Schwarzschild metric) and could not exactly describe the spacetime of a black hole embedded in a Friedmann−Robertson−Walker (FRW) universe. Moreover, Eq. (11) is obtained by ignoring the backreaction of the phantom matter on the black hole metric. In a low matter density background, this effect can be safely ignored; but when the background density becomes large (e.g., comparable to the black hole density), the metric describing this black hole will be modified significantly. So, in order to study this issue, in principle, one should use new exact solutions of the Einstein equations describing dynamical black holes embedded in an FRW universe driven by phantom energy and accreting this cosmic fluid. This issue has been addressed in Ref. [16].

In Ref. [16], the authors considered the generalized McVittie metric,

$$\begin{aligned} ds^2 \;=\; & -\frac{\left[1 - \frac{M(t)}{2a(t)r}\right]^2}{\left[1 + \frac{M(t)}{2a(t)r}\right]^2} dt^2 + a^2(t)\left[1 + \frac{M(t)}{2a(t)r}\right]^4 \\ & \times\left(dr^2 + r^2 d\Omega^2\right), \end{aligned} \tag{15}$$

in the background of an imperfect fluid with a radial heat flux and, possibly, a radial mass flow simulating accretion onto a

[3] This point can be easily understood. In [9], the full formulation of the fluid accretion onto a black hole is fulfilled within the framework of conventional general relativity. As aforementioned, the usual general relativity is also suitable to describe the quantum cosmology provided that all the quantum gravity effects could be effectively attributed to the energy-momentum tensor, i.e. $G_{\mu\nu} = 8\pi T^{\rm eff}_{\mu\nu}$. Hence, under such circumstances, the theory of fluid accretion onto black hole can also be used in the cyclic model of universe provided that ρ_∞ and $p(\rho_\infty)$ in Eq. (11) are replaced by $\rho_{\rm eff}$ and $p_{\rm eff}$. By far, we have shown that the relation $\dot{M} \sim -M^2\dot{H}$ may still hold for the case of cyclic model.

[4] In this case, the phantom dominated universe has the scale factor as $a(t) = T^{2/[3(1+w)]}$, provided that w is a constant smaller than -1. Here $T = a_{\rm i}^{3(1+w)/2} + 3(1 + w)/2\sqrt{1 - \Omega^0_{\rm m}}H_0(t - t_{\rm i})$, in which $a_{\rm i}$ and $t_{\rm i}$ are the initial values for the scale factor and time, respectively, at the onset of phantom domination, and $\Omega^0_{\rm m}$ and H_0 are respectively the fractional matter density and Hubble parameter of today. Hence, for this case, we have $C(t) = 12\pi A(1 - \Omega^0_{\rm m})H_0^2(|w| - 1)(t - t_{\rm i})a_{\rm i}^{3(|w|-1)/2}T^{-1}$.

black hole embedded in an FRW universe. The imperfect fluid is described by the stress-energy tensor

$$T_{\mu\nu} = (p+\rho)\, u_\mu u_\nu + p g_{\mu\nu} + q_\mu u_\nu + q_\nu u_\mu, \tag{16}$$

where $u^\mu = \left(|g_{00}|^{-1/2}, 0, 0, 0,\right)$ is the fluid four-velocity and $q^\mu = (0, q, 0, 0,)$ is the radial heat current. Under this situation, it was shown in [16] that the appropriate notion of mass for a cosmological black hole is the Hawking−Hayward quasilocal mass,

$$m_H(t) = M(t) = M_0 a(t). \tag{17}$$

Obviously, this mass is always increasing in an expanding universe. In Ref. [17], the authors further showed that, under certain assumptions, only those with comoving Hawking−Hayward quasilocal mass are generic, in the sense that they are late-time attractors. Therefore, we see that if the backreaction is considered in the calculation of the accretion of fluid, the conclusion will be opposite to that of Babichev et al., i.e., the physical black hole mass may instead increase due to the accretion of phantom energy. If this is the case, the black hole will certainly not be torn up by phantom energy even in a usual phantom dominated universe. This implies that this subject is highly speculative, and there exist various possibilities that cannot be excluded or ignored.

In the present work, we are restricted to the framework of Ref. [9]. To avoid misleading readers, we must admit the drawbacks of this framework, as discussed above. In this framework, the cosmic fluid, namely the phantom energy, is treated as a test fluid, so the analysis does not have much to say about the destruction of black holes in a phantom-dominated universe, which is a markedly different physical situation. However, actually, we are currently lacking a satisfactory method to deal with such a subject. The method in Ref. [16] is also highly speculative, for example, the metric solution (15) is also hypothesized but not derived exactly. Under the circumstances, we can but using the means currently available to tentatively draw some conclusions on this topic.

In addition, it should be mentioned that in Ref. [16] another interesting possibility was also discussed. In that case, because the future universe is dominated by phantom dark energy, the black hole apparent horizon and the cosmic apparent horizon will eventually coincide and, after that, the black hole singularity will become naked in finite time, violating the cosmic censorship conjecture. Therefore, in our case, a question naturally arises asking whether the cosmological horizon can be smaller than the black hole horizon during the period of interest. Now we shall discuss this issue.

In any case we can rewrite Eq. (14) as

$$r_{\rm sch} = \frac{r^{(i)}_{\rm sch}}{1 + \sqrt{2\pi} A H(t) r^{(i)}_{\rm sch}}, \tag{18}$$

where $r^{(i)}_{\rm sch}$ is the initial Schwarzschild radius. The constant A is of order unity, and in Ref. [9] it is determined, $A = 4$. Today the universe is in the low energy regime, we have $H(t) \sim 0$, so it is clear that $r_{\rm sch} = r^{(i)}_{\rm sch}$. However, when the universe goes into the high energy regime, the Hubble parameter $H(t)$ becomes very large; consequently we have $r_{\rm sch} \sim (\sqrt{2\pi}A)^{-1} H^{-1}$. In other words, when the Hubble parameter becomes enormously large, the Hubble radius will be one order of magnitude larger than the Schwarzschild black hole radius,

$$H^{-1} \sim 10 r_{\rm sch}. \tag{19}$$

That is to say, the Hubble volume at least involves about 10^3 black hole volume at any stages. Note that when H^{-1} diminishes, $r_{\rm sch}$ also diminishes. In the ordinary phantom cosmology, there is a "big-rip" singularity where $H \to \infty$ (so $H^{-1} \to 0$), so that $r_{\rm sch} \to 0$ when the universe approaches the big-rip. This is the essential of Ref. [9]. However, in our case, H^{-1} first decreases ($0 < x < 1/2$) and then increases ($1/2 < x < 1$); so $r_{\rm sch}$ will also first decrease and then increase until restoring its initial value $r^{(i)}_{\rm sch}$ at the turnaround point ($x = 1$).

Through the above analysis, we show that there may exist another possibility, i.e., black holes cannot be torn apart by phantom dark energy in this cyclic universe. The analysis is based on the assumption that the change rate of black hole mass is determined by the effective energy density $\rho_{\rm eff}$ and the effective pressure $p_{\rm eff}$. The reason of considering such a possibility is due to the inspiration of the models of quintom cyclic universe (or "quintom bounce") [18]. Imagining the universe filled with quintom dark energy with the energy density ρ behaving like the effective quantity $\rho_{\rm eff}$ of this paper, this quintom matter obviously can cause the cyclicity of the universe through the usual Friedmann equation $H^2 = (8\pi/3)\rho$. In this situation, through the accretion of quintom matter, the black hole mass change rate is determined by $\dot{M} = 4\pi A M^2(\rho + p)$ indicating that the black hole mass will first decrease and then increase. The model discussed in this paper is indistinguishable from the quintom cyclic model [18] on the cosmological evolution. So, inspired by this fact, we consider the possibility that in the process of fluid accretion onto a black hole, the effective quantities might play a crucial role.

Of course, we cannot indiscreetly claim that this is a definitive conclusion. But we cannot exclude this possibility either. Perhaps one might argue that $\rho_{\rm eff}$ and $p_{\rm eff}$ are not proper physical quantities, however, the possible mechanism discussed in this paper provides a novel angle of view to ponder upon this subject. Taking this view into account, a similar application in braneworld cosmology has also been discussed in [19].

In summary, we discussed in this paper the problem of whether black holes can be torn apart by phantom dark energy in cyclic cosmology. For the cyclic cosmology resting on phantom dark energy described by (1), it has been viewed that the black hole problem can be resolved in the light of the characteristic of phantom. However, we have demonstrated in this paper that there may exist another possibility. Using the theory of fluid accretion onto black holes, we analyzed this possibility in detail and showed that the masses of black holes might first decrease and then increase, through phantom accretion onto black holes in the expanding stage of the cyclic universe. Though the conclusion sounds somewhat counterintuitive, this possibility cannot be excluded. The aim of this paper is to provide a novel angle of view to this profound topic.

Acknowledgments

The author acknowledges Rong-Gen Cai, Miao Li, Yun-Song Piao and Zhi-Guang Xiao for helpful discussions. This work was supported by the Natural Science Foundation of China (Grant No. 10705041).

[1] R. C. Tolman, Phys. Rev. **38**, 1758 (1931); R. C. Tolman, *Relativity, Thermodynamics and Cosmology* (Oxford University Press, New York, 1934).
[2] J. Khoury, B. A. Ovrut, P. J. Steinhardt and N. Turok, Phys. Rev. D **64**, 123522 (2001) [hep-th/0103239]; P. J. Steinhardt and N. Turok, Science **296**, 1436 (2002); P. J. Steinhardt and N. Turok, Phys. Rev. D **65**, 126003 (2002) [hep-th/0111098]; L. A. Boyle, P. J. Steinhardt and N. Turok, Phys. Rev. D **70**, 023504 (2004) [hep-th/0403026]; P. J. Steinhardt and N. Turok, Science **312**, 1180 (2006) [astro-ph/0605173].
[3] M. G. Brown, K. Freese and W. H. Kinney, JCAP **0803**, 002 (2008) [arXiv:astro-ph/0405353].
[4] L. Baum and P. H. Frampton, Phys. Rev. Lett. **98**, 071301 (2007) [hep-th/0610213].
[5] X. Zhang, arXiv:0711.0667 [hep-th]; X. Zhang, Eur. Phys. J. C **59**, 755 (2009).
[6] R. R. Caldwell, M. Kamionkowski and N. N. Weinberg, Phys. Rev. Lett. **91**, 071301 (2003) [astro-ph/0302506].
[7] A. Ashtekar, T. Pawlowski and P. Singh, Phys. Rev. D **73**, 124038 (2006) [gr-qc/0604013]; A. Ashtekar, T. Pawlowski and P. Singh, Phys. Rev. D **74**, 084003 (2006) [gr-qc/0607039]; A. Ashtekar, AIP Conf. Proc. **861**, 3 (2006) [gr-qc/0605011]; P. Singh, K. Vandersloot and G. V. Vereshchagin, Phys. Rev. D **74**, 043510 (2006) [gr-qc/0606032].
[8] Y. Shtanov and V. Sahni, Phys. Lett. B **557**, 1 (2003) [gr-qc/0208047].
[9] E. Babichev, V. Dokuchaev and Yu. Eroshenko, Phys. Rev. Lett. **93**, 021102 (2004) [gr-qc/0402089].
[10] P. F. Gonzalez-Diaz, Phys. Rev. Lett. **93**, 071301 (2004) [astro-ph/0404045].
[11] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999) [hep-ph/9905221]; L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999) [hep-th/9906064].
[12] M. Bojowald, Phys. Rev. Lett. **100**, 221301 (2008) [arXiv:0805.1192 [gr-qc]].
[13] R. R. Caldwell, Phys. Lett. B **545**, 23 (2002) [astro-ph/9908168].
[14] B. Feng, X. L. Wang and X. M. Zhang, Phys. Lett. B **607**, 35 (2005) [astro-ph/0404224].
[15] X. Zhang and Y. Ling, JCAP **0708**, 012 (2007), [arXiv:0705.2656 [gr-qc]].
[16] C. Gao, X. Chen, V. Faraoni and Y. G. Shen, Phys. Rev. D **78**, 024008 (2008) [arXiv:0802.1298 [gr-qc]].
[17] V. Faraoni, C. Gao, X. Chen and Y. G. Shen, Phys. Lett. B **671**, 7 (2009) [arXiv:0811.4667 [gr-qc]].
[18] H. H. Xiong, Y. F. Cai, T. Qiu, Y. S. Piao and X. M. Zhang, Phys. Lett. B **666**, 212 (2008) [arXiv:0805.0413 [astro-ph]].
[19] P. Martin-Moruno, Phys. Lett. B **659**, 40 (2008) [arXiv:0709.4410 [astro-ph]].